Intelligent Control of a Tractor-Implement System Using Type-2 Fuzzy Neural Networks

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Abstract—Automatic guidance of agricultural vehicles would lighten the job of the operator, while accuracy is needed to obtain an optimal yield. Accurately navigating a tractor consists of controlling different dynamic subsystems (steering and speed). Instead of modeling the subsystem interaction prior to model-based control, we have developed a control algorithm which learns the interactions on-line from the measured feedback error. In this approach, a PD controller is working in parallel with a type-2 fuzzy neural network. While the former ensures the stability of the related subsystem, the latter learns the system dynamics and becomes the leading controller. In this study, two combinations of a PD controller with a type-2 fuzzy neural network are implemented: one for the yaw dynamics and one for the traction dynamics. The interactions between these subsystems are thus not taken into account explicitly, but considered as disturbances to be handled by the subsystem controllers. A novel sliding mode control theory-based learning algorithm is used to train the type-2 fuzzy neural networks, and the convergence of the parameters is shown by using a Lyapunov function.

I. INTRODUCTION

If the impact from other units to a subsystem is known, model-based approaches can be used to model the interactions between the mechatronic subsystems. However, most of the time, the interactions between mechatronic subsystems are not so clear such that they cannot be easily modeled. In such cases, the use of model-free techniques such as fuzzy logic theory, artificial neural networks (ANNs) and fuzzy neural networks (FNNs) can be a better option. For this purpose, the fuzzy, ANN and fuzzy neuro controllers have to be made adaptive/self-learning. Sliding mode control (SMC)-based learning algorithms can not only make the overall system more robust but also ensure faster convergence than the traditional learning techniques in online tuning of ANNs and type-1 FNNs (T1FNNs). There are various studies in the literature that aim to use the robustness property of SMC in the learning process of ANNs and T1FNNs [1], [2]. Conversely, the robustness and the stability properties of soft computing-based control strategies can also be analyzed through the use of SMC theory [3].

In this paper, an SMC theory-based learning algorithm is proposed to train type-2 fuzzy neural networks (T2FNNs) using type-2 fuzzy triangular membership functions. The motivation behind using a triangular membership function is the simplicity of this form among the other types of membership functions. It is to be noted that the distribution of the grades of membership between the lower and upper boundaries for a triangular membership function is linear [4]. The proposed learning algorithm establishes a sliding motion in terms of the T2FNN parameters, leading the learning error toward zero. The convergence of the algorithm is established and the stability conditions are given. As compared to the gradient-based learning methods which aim to minimize an error function, the learning parameters are tuned by the proposed algorithm in a way to enforce the error to satisfy a stable equation.

The main body of the paper contains five sections: In section II, the mathematical model of a tractor-implement system is given. In Section III, the proposed sliding mode feedback-error-learning approach is presented, and the parameter update rules for T2FNNs are proposed for the case of triangular membership functions. In Section IV, the simulation results are given. Finally, in Section V, conclusions are presented.

II. THE MATHEMATICAL MODEL OF A TRACTOR-IMPLEMENT SYSTEM

A. Yaw Motion Dynamics

While the velocities and sideslip angles are presented in Fig. 1(a), the forces at different locations of the tractor-implement system are shown in Fig. 1(b). The lateral motion of a tractor for a constant forward velocity is written as follows:

\[ m^l (\dot{v}_c^l + \dot{u}_c^l \gamma^l) = F_{f, f}^l \sin \delta + F_{f, f}^l \cos \delta + F_{t, r}^l + F_{h}^l \]  \hspace{1cm} (1)

where \( m^l \), \( F_{f, f}^l \), \( F_{f, f}^l \), \( F_{t, r}^l \) and \( F_{h}^l \) represent the mass of the tractor, the traction and lateral forces on the front wheel of the tractor, the lateral forces on the rear wheel of the tractor and the hitch point, respectively.

The yaw motion of the tractor is written as follows:

\[ F_{f, f}^l \gamma^l = F_{f, f}^l (F_{f, f}^l \sin \delta + F_{f, f}^l \cos \delta) - F_{t, r}^l - F_{h}^l \]  \hspace{1cm} (2)

where \( F_{f, f}^l \), \( F_{t, r}^l \) and \( F_{h}^l \) are the distance between the front axle and the center of gravity of the tractor, the distance between the rear axle and the center of gravity of the tractor, the distance between the hitch point and the center of gravity of the tractor, respectively.
The small steering angle assumption is reasonable for the intended application of tracking a smooth trajectory. Equations (1) and (2) are re-written with the assumption of zero longitudinal force $F_{i,f}$ as follows:

$$m'(\dot{V}_c + u_c' \gamma') = F_{i,j} + F_{i,r} + F_{i,h}$$

$$\dot{V}_c' = \dot{t}_j F_{i,j} - \dot{t}_r F_{i,r} - \dot{t}_h F_{i,h} \tag{3}$$

The velocity of the center of gravity of the tractor can be written with respect to the velocity of the center of gravity of the implement by using the hitch point:

$$\dot{u}_i' = u_i' = \dot{u}_i \cos \lambda - (v_c' - l_h \gamma') \sin \lambda$$

$$\dot{v}_i' = v_i' = \dot{v}_i \sin \lambda + (v_c' - l_h \gamma') \cos \lambda - l_h \gamma' \tag{4}$$

The lateral acceleration can be written as follows:

$$\ddot{v}_i' = \dot{u}_i' \lambda + u_i' \dot{\lambda} + (v_c' - l_h \gamma') \dot{\lambda} \lambda - l_h \dot{\gamma}'$$

where $\lambda$ is the hitch point angle which is defined as $\dot{\lambda} = \gamma - \gamma'$. It will also be small due to the small steering angle assumption:

$$\dot{v}_i' = \ddot{u}_i' \lambda + \dot{u}_i' \dot{\lambda} + (v_c' - l_h \gamma') \dot{\lambda} \lambda - l_h \dot{\gamma}' \tag{5}$$

The lateral acceleration can be re-written considering a constant longitudinal velocity:

$$\ddot{v}_i' = \dot{u}_i' \lambda + (v_c' - l_h \gamma') \dot{\lambda} \lambda - l_h \dot{\gamma}' \tag{6}$$

The lateral motion of the implement is written as follows:

$$m'(\dot{v}_c' + u_c' \gamma') = F_{i,j} + F_{i,h} \tag{7}$$

where $m', \dot{v}_c', u_c', \gamma', F_{i,j}$ and $F_{i,h}$ represent the mass of the implement, the lateral acceleration of the implement, the longitudinal velocity of the implement, the yaw angle of the implement, the lateral forces on the rear wheel of the implement and the hitch point, respectively.

The yaw motion of the implement is written as follows:

$$\ddot{t}_i' = m' t_h' \left( v_c' + u_c' \gamma' \right) - \left( t_h' \dot{\theta} + l_h \dot{\gamma}' \right) F_{i,r} \tag{8}$$

where $t_h', l_h', \dot{\theta}$ and $F_{i,r}$ represent the distance between the rear axle and the center of gravity of the implement, the distance between the hitch point and the center of gravity of the implement, the yaw rate of the center of gravity of the implement, the moment of inertia of the implement, respectively.

The lateral motion of the implement is written again considering (7):

$$F_{i,h} = m' u_i' + \left( v_c' - l_h \gamma' \right) - l_h \dot{\gamma}' - F_{i,r} \tag{9}$$

The relationship of the forces between the tractor and the implement for the hitch point is written considering a small $\lambda$ as follows:

$$F_{i,h} = -F_{i,h} \tag{10}$$

Equations (1)-(2) are re-written considering (10) and (11):

$$m'(\dot{V}_c + u_c' \gamma') = F_{i,j} + F_{i,r} - \left( m' u_i' \lambda \right)
+ \left( v_c' - l_h \gamma' \right) \dot{\lambda} \lambda - l_h \dot{\gamma}' \tag{11}$$

The yaw motion of the implement is written considering (7) as follows:

$$\ddot{t}_i' = m' t_h' \left( v_c' + u_c' \gamma' \right) - \left( t_h' \dot{\theta} + l_h \dot{\gamma}' \right) F_{i,r} \tag{12}$$

The lateral tire forces are calculated by using a linear model which assumes these to be proportional to the slip angles in [5], [6] as follows:

$$F_{i,j} = C_{\alpha_{\lambda},j, \alpha_{\gamma}} \quad j = \{f, r\}, \quad k = \{t, i\} \tag{13}$$

where $C_{\alpha_{\lambda},j, k}$, $j = \{f, r\}, k = \{t, i\}$, represents the cornering stiffness of the tires of the tractor and towed implement system, respectively. Finally, the equations of motion of the system are written as follows [7]:

$$\ddot{v}_i' = m' u_i' \lambda + (v_c' - l_h \gamma') \dot{\lambda} \lambda - l_h \dot{\gamma}' \tag{14}$$

$$m' \ddot{t}_i' = m' t_h' \left( v_c' + u_c' \gamma' \right) - \left( t_h' \dot{\theta} + l_h \dot{\gamma}' \right) F_{i,r} \tag{15}$$

$$m' \ddot{t}_i' = m' t_h' \left( v_c' + u_c' \gamma' \right) - \left( t_h' \dot{\theta} + l_h \dot{\gamma}' \right) F_{i,r} \tag{16}$$

$$m' \ddot{t}_i' = m' t_h' \left( v_c' + u_c' \gamma' \right) - \left( t_h' \dot{\theta} + l_h \dot{\gamma}' \right) F_{i,r} \tag{17}$$
B. Traction Dynamics

A quarter vehicle dynamics model is used to describe the traction dynamics of the vehicle. In this approach, the system model is written as follows:

\[ I \dot{\omega} = -r F_t + T \]
\[ m \ddot{u} = F_t \]

(18)

where \( I, \omega, u, r, F_t \) and \( T \) represent the inertia moment of the wheels of the vehicle, the angular velocity of the wheel of the vehicle, the linear velocity of the wheel of the vehicle, the radius of the wheel of the vehicle, the traction force, the torque on the wheel of the vehicle, respectively. The schematic diagram of a wheel is shown in Fig. 2.

![Schematic diagram of a wheel](image)

Fig. 2: The schematic view of the wheel

In order to reflect the effect from the yaw dynamics to longitudinal dynamics, an extra term has been added to the model which is function of the yaw rate. This term always tries to decrease the longitudinal velocity. It has also a coefficient \( K_r \), which can be chosen arbitrarily. The interconnection term that describes the influence of the yaw dynamics of the tractor is added into (18) as follows:

\[ I \dot{\omega} = -r F_t + T - K_r r \gamma \]
\[ m \ddot{u} = F_t - K_r |\gamma| \]

(19)

where \( K_r \) represents the coefficient of the interaction term.

The traction force is written as follows:

\[ \mu(s_j, c) = \frac{F_{trac}}{F_{z,j}} \]

(20)

where \( \mu(s_j, c), F_{trac}, F_{z,j} \) represent the adhesion coefficient, the traction force on the tire and the nominal vertical force at wheel contact, respectively.

The longitudinal slip ratio is defined as follows:

\[ s = \begin{cases} \frac{\omega}{r} - 1 & \text{if } u > r \omega, u \neq 0 \text{ for braking} \\ 1 - \frac{\omega}{r \omega} & \text{if } u < r \omega, r \omega \neq 0 \text{ for driving} \end{cases} \]

where \( s, r, \omega \) represent the longitudinal slip ratio, the radius of the wheel and the angular velocity of the wheel, respectively.

The adhesion coefficient is written as follows:

\[ \mu(s) = \frac{2 \mu_p s_p s}{s_p^2 + s^2} \]

(21)

where \( \mu_p \) and \( s_p \) are the peak values for various road conditions.

III. The Adaptive Fuzzy Neuro Control Approach

A. The control scheme

The proposed control scheme is schematically illustrated in Fig. 3. This method was originally proposed in [8] for robot control in which a neural network works in parallel with a PD controller. The arrows in Fig. 3 indicate that the outputs of the PD controllers are used to tune the parameters of the T2FNNs.

![Block diagram of the proposed adaptive fuzzy neuro scheme](image)

Fig. 3: Block diagram of the proposed adaptive fuzzy neuro scheme

B. Type-2 fuzzy neural network with triangular membership functions

The mathematical expression for the triangular membership function is expressed as:

\[ \mu(x) = \begin{cases} 1 - \frac{|x - c|}{d} & |x - c| < d \\ 0 & \text{otherwise} \end{cases} \]

(22)

where \( c \) and \( d \) are the center and the width of the membership function, \( x \) is the input vector. In Figs. 4(a) and 4(b), triangular type-2 fuzzy membership functions with uncertain width and uncertain center are shown. In this study, a triangular type-2 fuzzy membership function with uncertain width and fixed center (Figs. 4(a)) is considered.

Each membership function in the antecedent part is represented by an upper and a lower membership function. They are denoted as \( \overline{\mu}(x) \) and \( \underline{\mu}(x) \). The strength of the rule \( R_{ij} \) is obtained as a \( T \)-norm of the membership functions in the premise part (by using a multiplication operator):

\[ W_{ij} = \overline{\mu}_i(x_1) \underline{\mu}_j(x_2) \]

(23)

\[ \overline{W}_{ij} = \overline{\mu}_i(x_1) \overline{\mu}_j(x_2) \]

(24)
The triangular membership functions $\mu_{ij}(x_1)$, $\mu_{ij}(x_2)$, $\mu_{2j}(x_1)$, and $\mu_{2j}(x_2)$ of the inputs $x_1$ and $x_2$ in the above expression have the following appearance:

$$\mu_{ij}(x_1) = \begin{cases} 1 - \frac{x_1 - c_{ij}}{d_{ij}} & |x_1 - c_{ij}| < d_{ij} \\ 0 & \text{otherwise} \end{cases}$$  \hspace{1cm} (25)

$$\mu_{ij}(x_2) = \begin{cases} 1 - \frac{x_2 - c_{ij}}{d_{ij}} & |x_2 - c_{ij}| < d_{ij} \\ 0 & \text{otherwise} \end{cases}$$  \hspace{1cm} (26)

$$\mu_{2j}(x_1) = \begin{cases} 1 - \frac{x_1 - c_{2j}}{d_{2j}} & |x_1 - c_{2j}| < d_{2j} \\ 0 & \text{otherwise} \end{cases}$$  \hspace{1cm} (27)

$$\mu_{2j}(x_2) = \begin{cases} 1 - \frac{x_2 - c_{2j}}{d_{2j}} & |x_2 - c_{2j}| < d_{2j} \\ 0 & \text{otherwise} \end{cases}$$  \hspace{1cm} (28)

\section*{C. Interval type-2 A2-C0 Takagi-Sugeno-Kang (TSK) model}

The T2FLS considered in this paper is of interval type, and it uses type-2 triangular membership functions in the premise part and crisp numbers in the consequent part. This structure is called A2-C0 fuzzy system [9], and it is shown in Fig. 5.

The fuzzy if-then rule $R_{ij}$ of a zero-order type-2 TSK model with two input variables where the consequent part is a crisp number can be defined as follows:

$$R_{ij}: \text{If } x_1 \text{ is } \tilde{A}_{1i} \text{ and } x_2 \text{ is } \tilde{A}_{2j}, \text{ then } f_{ij} = d_{ij}$$  \hspace{1cm} (29)

The followings are the operations in each layer in Fig. 5:

\textbf{Layer 1}: The input signals feed the system. The related figure shows the system for two inputs which are the error and the time derivative of the error.

\textbf{Layer 2}: For each input signal entering the system, the membership degrees $\tilde{\mu}$ and $\tilde{\mu}$ are determined.

\textbf{Layer 3}: This layer calculates the firing strengths of the rules which are realized using the prod t-norm operator using (23) and (24).

\textbf{Layer 4}: This layer determines the outputs of the linear functions $f_{ij}$ ($i = 1, \ldots, I$ and $j = 1, \ldots, J$), in the consequent parts for the two inputs case.

$$f_{ij} = d_{ij}$$  \hspace{1cm} (30)

\textbf{Layer 5}: This layer computes the product of the membership degrees $W_{ij}$ and $\tilde{W}_{ij}$ and linear functions $f_{ij}$.

\textbf{Layer 6}: This layer includes two summation blocks. One of these blocks computes the sum of the output signals from layer 5 (the numerator part of (31)) and the other block computes the sum of the output signal of layer 3 (the denominator part of (31)).

\textbf{Layer 7}: This layer calculates the output of the network using (32).
The following assumptions have been used in this investigation:

Both the input signals \( x_1(t) \) and \( x_2(t) \), and their time derivatives can be considered bounded:

\[
|x_1(t)| \leq \tilde{B}_x, \quad |x_2(t)| \leq \tilde{B}_x \quad \forall t \quad (34)
\]

\[
|x_1(t)| \leq \tilde{B}_x, \quad |\dot{x}_2(t)| \leq \tilde{B}_x \quad \forall t \quad (35)
\]

where \( \tilde{B}_x \) and \( \tilde{B}_\dot{x} \) are assumed to be some known positive constants.

It is obvious that \( 0 < \tilde{W}_j < 1 \) and \( 0 < \tilde{W}_j < 1 \). In addition, it can be easily seen that \( \sum_{i=1}^{I} \sum_{j=1}^{I} \tilde{W}_{ij} = 1 \) and \( \sum_{i=1}^{I} \sum_{j=1}^{I} \tilde{W}_{ij} = 1 \).

It is also considered that, \( \tau \) and \( \dot{\tau} \) will be bounded signals too, \( i.e. \)

\[
|\tau(t)| < B_\tau, \quad |\dot{\tau}(t)| < B_\dot{\tau} \quad \forall t \quad (36)
\]

where \( B_\tau \) and \( B_\dot{\tau} \) are some known positive constants.

D. The sliding mode learning algorithm

Using the principles of SMC theory [10] the zero value of the learning error coordinate \( \tau_c(t) \) can be defined as a time-varying sliding surface, \( i.e. \)

\[
\tau_c(t, \tau) = \tau_c(t) = \tau_c(t) + \tau(t) = 0 \quad (37)
\]

which is the condition that the T2FNN is trained to become a nonlinear regulator to obtain the desired response during the tracking-error convergence movement by compensation for the nonlinearity of the controlled plant.

The sliding surface for the nonlinear system under control \( S_p(e, \dot{e}) \) is defined as:

\[
S_p(e, \dot{e}) = e + \chi e \quad (38)
\]

with \( \chi \) being a positive constant determining the slope of the sliding surface.

Definition: A sliding motion will appear on the sliding manifold \( S_c(t, \tau) = \tau_c(t) = 0 \) after a time \( t_h \), if the condition \( S_c(t)S_c(t) = \tau_c(t) \tau_c(t) < 0 \) is satisfied for all in some non-trivial semi-open subinterval of time of the form \( [t, t_h) \subset (0, t_h) \).

It is desired to devise a dynamical feedback adaptation mechanism, or online learning algorithm for the T2FNN parameters such that the sliding mode condition of the above definition is enforced.

E. The parameter update rules for T2FNN

The parameter update rules for a T2FNN which has two inputs are given in the following theorem.

Theorem 1: If the adaptation laws for the parameters of the considered T2FNN are chosen as:

\[
\dot{\alpha}_{ij} = \left\{ \begin{array}{ll}
\dot{\alpha}_{ij} = \dot{x}_1 & \text{if } x_1 - \alpha_{ij} < d_{ij} \\
0 & \text{otherwise}
\end{array} \right. \quad (39)
\]

\[
\dot{\alpha}_{ij} = \left\{ \begin{array}{ll}
\dot{\alpha}_{ij} = \dot{x}_1 & \text{if } x_1 - \alpha_{ij} < d_{ij} \\
0 & \text{otherwise}
\end{array} \right. \quad (40)
\]

then, given an arbitrary initial condition \( \tau_c(0) \), the learning error \( \tau_c(t) \) will converge firmly to zero during a finite time \( t_h \).

\[
\alpha = B_\tau \quad (49)
\]

The relation between the sliding line \( S_p \) and the zero adaptive learning error level \( S_c \) is determined by the following equation:

\[
S_c = \tau_c = k_D e + k_F \left( \dot{e} + k_F \right) = k_D S_p \quad (50)
\]

The tracking performance of the feedback control system can be analyzed by introducing the following Lyapunov function candidate:

\[
V_p = \frac{1}{2} \dot{S}_p^2 \quad (51)
\]

Theorem 2: If the adaptation strategy for the adjustable parameters of the T2FNN is chosen as in (39)-(47), then the negative definiteness of the time derivative of the Lyapunov function in (51) is ensured.

Proof: The reader is referred to Appendix B.

Remark: The obtained result means that, assuming the SMC task is achievable, using \( \tau_c \) as a learning error for the T2FNN together with the adaptation laws (39)-(47) enforces the desired reaching mode followed by a sliding regime for the system under control.
IV. SIMULATION STUDIES

Figure 6 shows the overall control scheme for the trajectory control of the tractor-implement system. The representation of the kinematic controller is as follows:

\[
\begin{align*}
    u_{\text{ref}} &= \sqrt{x_d^2 + y_d^2} + l_x \tanh \left( \frac{k_x}{l_x} e_x \right) \\
    \phi_{\text{ref}} &= \arctan \left( \frac{y_d}{x_d} \right) + l_y \tanh \left( \frac{k_y}{l_y} e_y \right)
\end{align*}
\]

(52)

where, \( e_x = x_d - x \) and \( e_y = y_d - y \) are the current position errors in the axes X and Y, respectively. The parameters \( k_x > 0 \) and \( l_x > 0 \) are the gains of the controller, \( l_x \in \mathbb{R} \) and \( l_y \in \mathbb{R} \) are the saturation constants, and \((x,y)\) and \((x_d,y_d)\) are the current and desired coordinates, respectively.

![Fig. 6: Block diagram of the control scheme](image)

The numerical values used in this study are \( m_l = 9391 \) kg, \( m_e = 2127 \) kg, \( l_x^2 = 35709 \) kg m\(^2\), \( l_y^2 = 6402 \) kg m\(^2\), \( l_x = 1.7 \) m, \( l_y = 1.2 \) m, \( l_\phi = 2.1 \) m, \( l_\phi^2 = 3.62 \) m, \( l_t = 0.1 \) m, \( C_{\alpha,e} = 220 \) KN rad\(^{-1}\), \( C_{\alpha,\phi} = 220 \) KN rad\(^{-1}\), \( C_{\alpha,t} = 167 \) KN rad\(^{-1}\). These numerical values are collected from a John Deere MFWD tractor (model 7930, Deere and Co., Moline IL) [7]. The numerical values for the kinematic controller are chosen as \( l_x = l_y = k_x = k_y = 0.1 \). The sampling period of the simulations is set to 0.01 s. The number of membership functions for the input 1 and input 2 is chosen as \( I = J = 3 \) for all the simulations.

The coefficients of the PD controller for the yaw motion dynamics are set to \( k_p = 0.2 \) and \( k_d = 0.001 \) by trial-and-error method. The learning rate for the T2FNN 1 \( \alpha = 5 \times 10^{-3} \) for the yaw motion dynamics. Similarly, the coefficients of the PD controller for the traction dynamics are set to \( k_p = 70000 \) and \( k_d = 20 \) by trial-and-error method. The learning rate for T2FNN 2 \( \alpha = 2 \) for the traction dynamics. The coefficient of the interaction model in (19) is set to \( K_i = 50000 \). During the simulations, all the weight matrices in both premise and consequent part of the rules are initialized randomly.

The following reference signals have been applied to the system:

\[
\text{Reference}(t) = \begin{cases} 
    x_d(t) = 0.03t \text{ m} \\
    y_d(t) = 30\sin(0.0005t) \text{ m}
\end{cases}
\]

(53)

The adhesion coefficient \( \mu \) is set to 0.6. In order to be able to simulate changing road conditions for the tractor, a noise signal is added to \( \mu \) which is equal to \( SNR = 10 dB \).

The maximum interaction between the wheels and the surface occurs when the peak value of the longitudinal slip is \( s_p = 0.2 \). Moreover, in order to be able to simulate the noise signal for the GPS system, a noise signal is added to the X and Y coordinates. In the following simulations, the results for \( SNR = 50 dB \) are shown.

As can be seen from Fig. 7, when the PD controllers act alone, the control performances of both the longitudinal dynamics and yaw dynamics are not reasonable. The fine tuning of such controllers in real life is challenging, because in addition to the interactions of the subsystems, there exist unmodeled dynamics and uncertainties in real world applications.

![Fig. 7: PD controllers working alone: (a) Yaw angle response (b) Longitudinal velocity response (c) Yaw angle error (d) Longitudinal velocity error](image)

Figure 8 shows the condition when the PD controllers are working in parallel with the T2FNNs in which the control performance of both longitudinal dynamics and yaw dynamics are improved. These results indicate that T2FNNs are able to learn the system dynamics after a while, and they can improve the overall performance of the system without the need of fine tuning the conventional controllers which is a very demanding process in real-life applications.

Figure 9 shows the control signals coming from the conventional PD controllers and T2FNNs. As can be seen from Fig. 9, at the beginning, the dominating control signals are the ones coming from the PD controllers. After a short time, the T2FNNs are able to take over the control, thus becoming the leading controllers.

Figure 10 represents the online tuning of the parameter \( q \) in (32) which weights the sharing of lower and upper firing levels of each fired rule. Instead of using a constant value for the parameter \( q \), it is shown that this parameter has been tuned using an SMC theory-based learning algorithm.
of learning the plant model online instead of using accurate predefined dynamical equations of the system. The use of the combination of fuzzy logic control, artificial neural networks and sliding mode control theory harmoniously allows us to better handle the interactions in the subsystems, uncertainties and lack of modeling information. The simulation studies show that when the T2FNN is used in parallel with a conventional PD controller, the overall system gives better performance in terms of a smaller settling time and zero steady state error. In addition to its robustness, another prominent feature is the computational simplicity of the proposed approach. Encouraged by these simulation results, an experimental investigation is currently in progress.

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APPENDIX A

Proof of Theorem 1:

The time derivatives of (25) and (28) are as follows:

\[
\dot{\mu}_{1i}(x_1) = \left(-\frac{x_1 - \bar{c}_{1i}}{d_{1i}} + \frac{x_1 - c_{1i}}{d_{1i}^2}d_{1i}\right)\text{sgn}\left(\frac{x_1 - c_{1i}}{d_{1i}}\right)
\] (54)

\[
\mu_{1i}(x_1) = \left(-\frac{x_1 - \bar{c}_{1i}}{d_{1i}} + \frac{x_1 - c_{1i}}{d_{1i}^2}d_{1i}\right)\text{sgn}\left(\frac{x_1 - c_{1i}}{d_{1i}}\right)
\] (55)

\[
\dot{\mu}_{2j}(x_2) = \left(-\frac{x_2 - \bar{c}_{2j}}{d_{2j}} + \frac{x_2 - c_{2j}}{d_{2j}^2}d_{2j}\right)\text{sgn}\left(\frac{x_2 - c_{2j}}{d_{2j}}\right)
\] (56)

\[
\mu_{2j}(x_2) = \left(-\frac{x_2 - \bar{c}_{2j}}{d_{2j}} + \frac{x_2 - c_{2j}}{d_{2j}^2}d_{2j}\right)\text{sgn}\left(\frac{x_2 - c_{2j}}{d_{2j}}\right)
\] (57)

V. CONCLUSIONS

The novel aspect of this paper is the use of an SMC theory-based learning algorithm to train the parameters of the T2FNNs. The proposed control scheme is tested on a tractor model which has two subsystems in interacting with each other. For each subsystem, the control structure proposed consists of a PD controller and a T2FNN which is capable
\[
A = \left( \frac{x_1 - c_{1i}}{d_{1i}} \right) d_{1i} - \left( \frac{x_1 - c_{1j}}{d_{1j}} \right) d_{1j} - \left( \frac{x_2 - c_{2j}}{d_{2j}} \right) sgn(E_2) (1 - E_2 sgn(E_2)) \\
+ \left( \frac{x_2 - c_{2j}}{d_{2j}} \right) d_{2j} - \left( \frac{x_2 - c_{2j}}{d_{2j}} \right) sgn(E_2) (1 - E_2 sgn(E_2)) \\
= -2a sgn(\tau_e) \mu_{1j} \mu_{2j} \\
= -2a sgn(\tau_e) W_{ij}
\]

\[
\dot{W}_{ij} = -2a \dot{\tilde{W}}_{ij} sgn(\tau_e) - \frac{(\tilde{W}_{ij}) \sum_{j=1}^{J} \sum_{i=1}^{I} (2a \tilde{W}_{ij} sgn(\tau_e))}{(\sum_{i=1}^{I} \sum_{j=1}^{J} W_{ij})}
\]

By using the following Lyapunov function, the stability condition is checked as follows:

\[
V_c = \frac{1}{2} \dot{\epsilon}^2(t)
\]

\[
\dot{V}_c = \tau_c (\dot{\epsilon} + \epsilon (1 + \epsilon) + \epsilon \epsilon (1 + \epsilon))
\]

\[
\tau_n = q \sum_{i=1}^{I} \sum_{j=1}^{J} (f_{ij} \tilde{W}_{ij} + f_{ij} \dot{\tilde{W}}_{ij}) + (1 - q) \sum_{i=1}^{I} \sum_{j=1}^{J} (f_{ij} \tilde{W}_{ij} + f_{ij} \dot{\tilde{W}}_{ij})
\]

\[
\dot{\tau}_n = q \sum_{i=1}^{I} \sum_{j=1}^{J} (f_{ij} \dot{\tilde{W}}_{ij} + f_{ij} \dot{\tilde{W}}_{ij}) + (1 - q) \sum_{i=1}^{I} \sum_{j=1}^{J} (f_{ij} \dot{\tilde{W}}_{ij} + f_{ij} \dot{\tilde{W}}_{ij})
\]

If Eqs. (47) and (48) are inserted to the equation above, the following can be obtained:

\[
\dot{V}_c = \tau_c \dot{\epsilon} (\dot{\epsilon} - 2a sgn(\tau_e) + \dot{\epsilon}) < \left( -2a |\tau_e| + |\tau_e| |\beta_e| < 0 \right) \]

APPENDIX B

Proof Theorem 2: Evaluating the time derivative of the Lyapunov function in (51) yields:

\[
\dot{V}_p = \dot{S}_p S_p = \frac{1}{k_p S_p} S_p \leq \frac{|\tau_e|}{k_p} (\tilde{\alpha} + B_e) < 0, \quad \forall S_e, S_p \neq 0
\]

REFERENCES


